COMPARISON OF FINITE-DIFFERENCE AND ANALYTIC MICROWAVE CALCULATION METHODS

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ABSTRACT

Normal modes and power absorption distributions in microwave cavities containing lossy dielectric samples were calculated for problems of interest in materials processing. The calculations were performed both using a commercially available finite- difference electromagnetic solver and by numerical evaluation of exact analytic expressions. Results obtained by the two methods applied to identical physical situations were compared. Our studies validate the accuracy of the finite-difference electromagnetic solver. Relative advantages of the analytic and finite-difference methods are discussed.

INTRODUCTION

Use of exact analytic expressions provides accurate and relatively fast results for electromagnetic problems with sufficiently simple geometries. Numerical ***electromagnetic solvers ean*** treat complex geometries, but with less certain accuracy. The cooperative effort discussed in this paper applied and compared both methods to the representative problem of a lossy ceramic rod situated along the axis of a cylindrical cavity reactor to validate the numerical approach, and to begin a longer-term effort to take advantage of both approaches (and possibly also of approximate analytic methods) in solving problems related to microwave processing of materials,

DESCRIPTION OF THE STEADY-STATE MODEL

The relatively simple model consists of a closed metal cylindrical cavity reactor with an alurnina-like ceramic rod positioned along the entire length of the axis of symmetry. The cavity radius, ρ_c=4.69 cm, and length, 6.63 cm, correspond to a 2.45 GHz resonance frequency for the empty cavity TM010 mode. Rods with several different values of radius were modeled.

DESCRIPTION OF THE ANALYTIC METHOD

Electromagnetic properties of the microwave cavity reactor represented in the model described above were treated by analytic methods at JPL [1,2]. To account for radial variations of the complex dielectric constant the lossy dielectric sample (the rod) is partitioned into thin cylindrical zones, or shells, each of which is assumed to have uniform properties. The vacuum (or au) space around the rod is one additional zone, with relative dielectric constant of unity. The curved cavity wall may also be taken as one zone in order to include wall losses. Treated as a perfect electrical conductor, that zone is omitted and an appropriate boundary condition is imposed.

Maxwell's equations can be solved exactly for this shell model using a 4x4 matrix formalism originally due to Sphicopoules, Bernier, and Gardiol [3]. The normal mode frequencies are calculated as complex-valued roots of a complex-valued determinant, using a root finder developed at JPL. Ordinary matrix methods were then used to determine certain expansion coefficients that occur in expressions for normal mode fields. The formulas for this procedure provide rapid and efficient means for evaluating normal mode electric and magnetic fields and power absorption density.

In the exact solution method just described, the flat end plates are treated as perfect electrical conductors. If desired, microwave power absorption in the end plates can be calculated from the adjacent magnetic fields and a surface resistance approximation. The end-plate losses can then be combined with the results of the exact solution (assuming lossless end plates) to calculate the Q of the cavity that includes all the losses. In addition, the field strengths inside the cavity are renormalized so that they correspond to a specified value of total power dissipated in the sample and cavity walls, including the end plates,

DESCRIPTION OF THE FINITE-DIFFERENCE CODE

The finite-difference code used in this study was the frequency domain module for modes of resonance contained *within* the electomagnetic/plasma physics code known as MAFIA (for "MAxwell's equations by Finite-Integral Algorithm"). Maxwell's equations are transformed into a set of fully self-consistent discrete matrix equations and solved (without sources). Two- or three-dimensional geometries may be treated With the latest release (MAFIA 3.20), no-loss, low-loss, and lossy solvers are all available for solving for the resonance frequency and the field patterns of the modes. The solvers which treat lossy dielectrics account for finite conductivity. The post-processor is also able to compute the approximate losses from the no-loss field solution, given the relevant material properties. Skin-effect losses in conducting walls are evaluated using the magnetic field adjacent and parallel to the. wall surface, as in the treatment of end walls by the analytic method,

Time domain modules that account for sources of microwave power are also available within MAFIA. An extension of these modules that will treat nonlinear (temperature-dependent) materials is now being tested by the code developer, Computer Simulation Technology (CST) in Darmstadt, Germany.

COMPARISON OF RESULTS

The frequency vs. normalized rod radius for the TM010 and TM110 modes using the two methods are compared in Fig. 1. For the numerical approach, a 100X100 uniform rectangular mesh was used. The agreement is excellent, the maximum deviation being about 0.1%. Fig. 2 shows the comparison of results from the two methods for quality factor due to losses in the dielectric using the TM010 mode. Similarly excellent agreement was obtained with the TM110 mode.

In Fig, 3, results are compared for power density vs. radius with three different rod sizes, using the TM010 mode. Fig. 4 shows equivalent results for two rod sizes for the TM110 mode.

Table I compares results for the lower frequency Hybrid-l 11 mode. For this case the relative dielectric constant is 8.5459 and the loss tangent is 0.0005S9. With such a low loss case, the results from the lossless and lossy solvers are found to be in good agreement, as expected.

Table I. Comparison of Results for a Hybrid-1 11 Mode

APPROACH	REAL FREQUENCY	QUALITY FACTOR
analytic	1.8759 GHz	2.5764
numeric, lossless	1.8781 (.13% high)	2608.0 (1.2% high)
numeric, lossy	1.8781 (.13% high)	2620.7 (1.7% high)

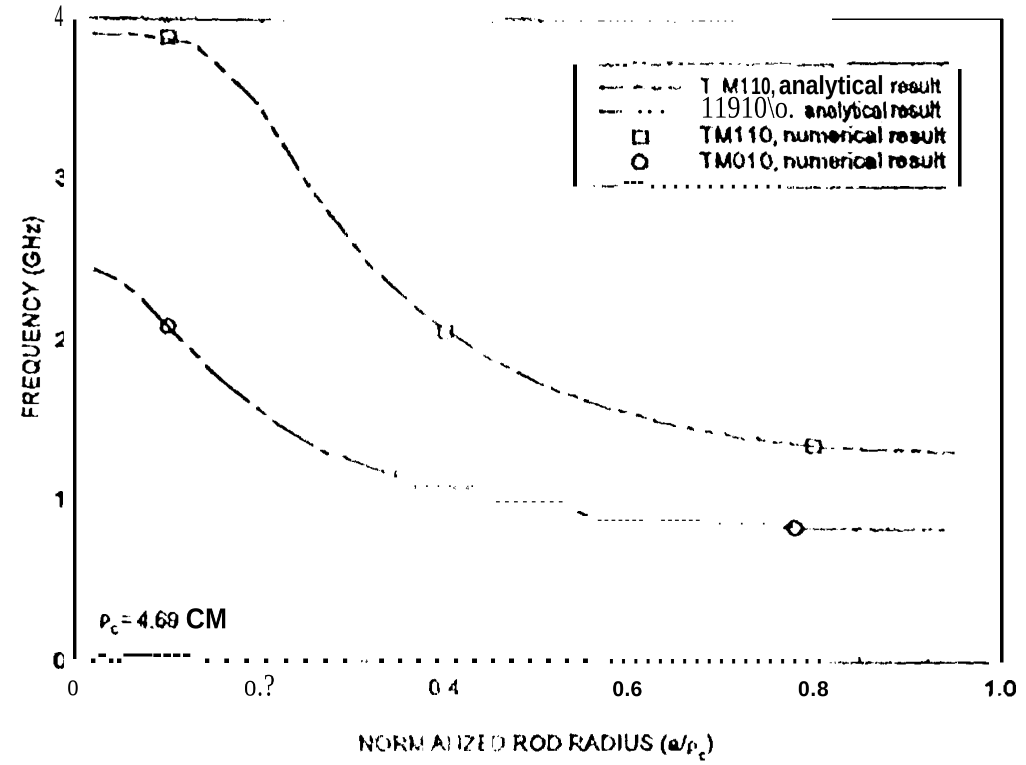

Figure 1. Resonance frequency vs. normalized rod radius for TM010 and TM110 modes.

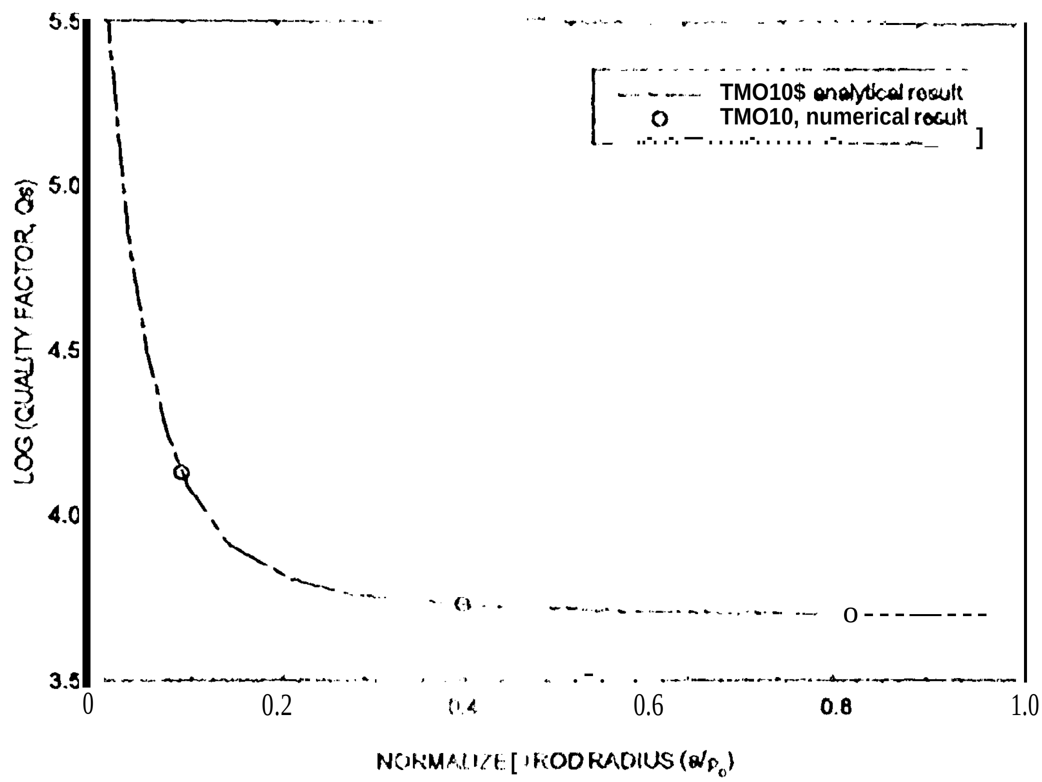

Figure 2. LOG10(Quality Factor) vs. normalized rod radius for TM010 mode.

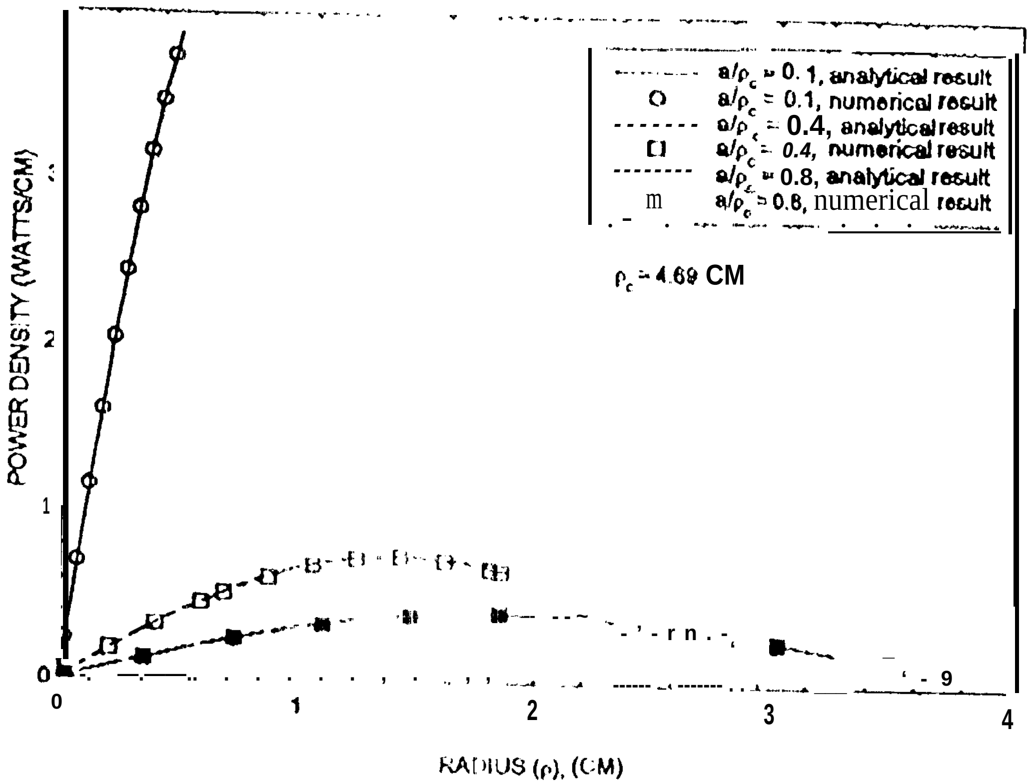

Figure 3. TM01 O mode power absorption profile for various rod radii, with 1 watt absorbed.

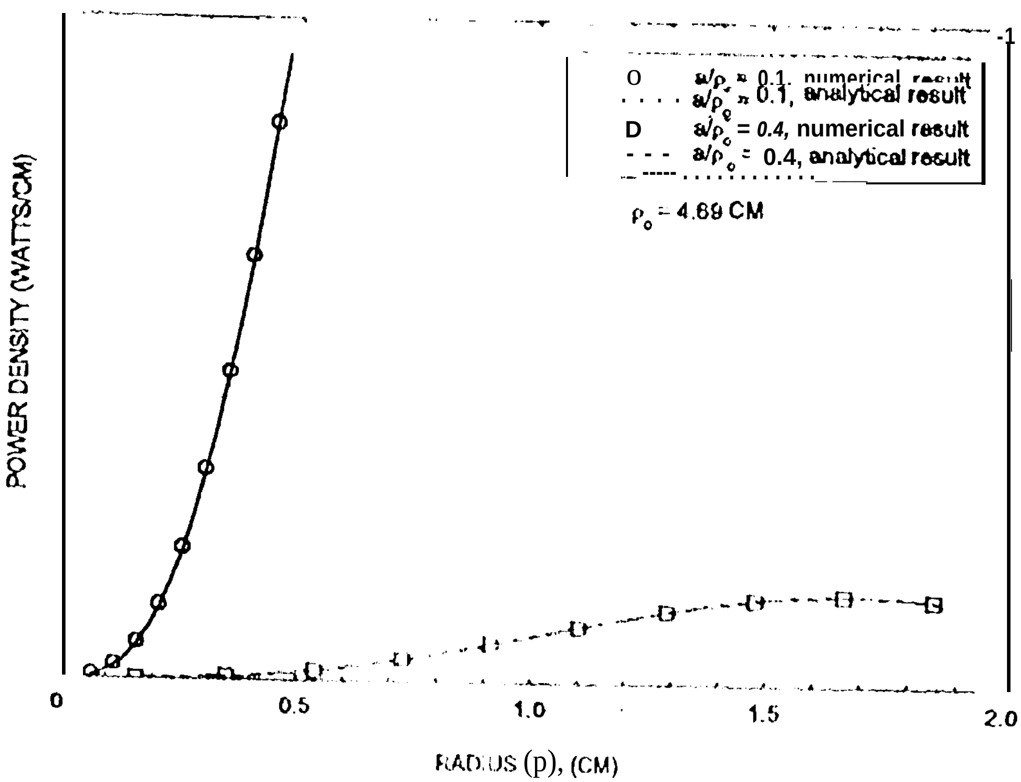

Figure 4. TM] 10 mode power absorption profile for various rod radii, with 1 watt absorbed.

Table II. Comparison of Results for TM010 Mode Loss Tangent=0.4

APPROACH	REAL FREQUENCY	QUALITY FACTOR
analytic	1.0146 GHz	2.7740
numeric, lossless	1.0683 (5.2% high)	2.7036 (2.5% low)
numeric, lossy	1.0177 (.31% high)	2.7749 (.03% high)

Table II compares results for the TM010 mode with a relatively high loss tangent, 0.4. With such high loss the results from the lossy solver should be more accurate than those from the lossless solution. The comparison of results from the numeric approach to those from the analytic approach confirms this assumption.

Table III compares results for the TM010 mode with linearly varying complex dielectric constant as a function of radius. Forty shell regions were used to represent the rod with both approaches. The relative real dielectric constant varies from 15 on axis to 10 at the surface; while the imaginary part varies from 0.75 to 0.5. This choice results in a constant loss tangent of 0.05. The excellent agreement between the calculation methods validates the approaches for treating a spatially varying dielectric constant.

Table III. Comparison of Results for TM010 Mode, Linearly Varying Dielectric Constant

APPROACH	REAL FREQUENCY	QUALITY FACTOR
analytic	.93252 GHz	21.079
numeric, lossless	.93179 (.078% low)	21.053 (.12% low)

The good-to-excellent agreement typical of the results provides mutual validation of the two basic approaches. Either method can provide detailed results for parametric variations, even for quite lossy materials. Since the analytic model gives exact results, the differences between the two approaches due to different sample sizes or properties, modal field patterns and/or frequencies can be evaluated. This is very beneficial to adjusting numerical modeling parameters to best obtain desired accuracy with more complex geometries. For example, results for the quality factor due to dielectric loss in a rod with a/ρ_c=0.4 deviated by 0.16% using the TM010 mode (and still less for the TM020 mode), when the mesh was changed from 100x100 to 50x50. Results for the 100x100 mesh had been validated by the analytic approach. However, the deviation from the validated results jumped to 0.50% using a 30x30 mesh. Similarly, the frequency of the two modes changed less than 0.1% going to a 50x50 mesh, but the deviation for the TM020 mode frequency jumped to 0.48%. for a 25x25 mesh.

ADVANTAGES OF THE ANALYTIC APPROACH

The analytic approach can typically provide highly accurate solutions to simple electromagnetic problems much faster than numerical methods such as the finite-difference approach. This makes extensive parametric studies more economically feasible. The exact formulas provide a ready means for detailed evaluation of the solution, as desired. This approach is also better adapted to solving problems where both electromagnetic and thermal properties should be calculated self-consistently, e.g. when realistic thermal emissivities of both the sample and the cavity wall are required in the model.

ADVANTAGES OF THE NUMERICAL APPROACH

Whereas the analytic approach is limited to problems with relatively simple geometry and symmetry, the numerical approach considered here is limited primarily by the ability of a computer to solve the problem discretized on a mesh, and to accomplish this in a "reasonable" time. This means that the overall problem size should not be extremely large compared to the smallest detail (including electromagnetic field variations as well as geometrical model variations) that must be resolved on the mesh. With that limitation geometries can be quite complex. The multi-solver numerical code is also capable of solving many aspects of complex problems and a great variety of simple problems. For example, in the analytic approach for a cavity reactor a source of excitation cannot be readily included in the model, whereas the time-domain numerical code solver can provide the multi-mode solution for a reactor large compared to the wavelength of the drive signal, and even permit the choice of pulsed, transient, or sinusoidal microwave drive.

BENEFITS OF TWO-METHOD APPROACH TO SOLVING REAL PROBLEMS

Through appropriate application of each of the methods described, the advantages of both methods can be exploited. For example, parametric studies of a simple model can be rapidly performed using the analytic approach, and an approximate optimum set of parameters extracted from the results, This can then be used as a good point-of-departure design for rigorous analysis using the numerical approach. in general, this will result in better designs achieved in a shorter time. For solution of problems of unusual nature or material properties, it is valuable to start with mutual validation of the two approaches by applying them to a representative, sufficiently simplified version of the problem, In this way any significant errors in either approach maybe detected.

APPLICATION TO PROBLEMS WITH CYLINDRICAL GEOMETRY

In general, the two-method approach is useful for validation of new variations in method, mutual validation of analytic and numerical solutions to specific problems, and evaluation of numerical error. The following is a representative list of possible problems for using the two-method approach discussed above.

1. Dielectric constant measurements for a large range of sample sizes and a large range of lossy dielectric materials (not limited to small sample. size and low-loss materials needed for cavity perturbation theory).
2. inverted temperature profile in materials processing.
   a. chemical vapor filtration
   b. improved grain characteristics for high critical current in high Tc superconductors.
3. Fiber coating: determining optimum experimental processing conditions for heating the fiber as its diameter increases.
4. Uniform heating of a fluid flowing in a cylindrical tube..

FUTURE JOINT EFFORTS

The following is a list of joint efforts that are planned or under consideration: (1) comparison of analytic and finite-difference results for transient conditions, (2) validation of a normal mode expansion method, particularly for practical application to batch microwave processing, and (3) application to the design of a batch-processing reactor.

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